

Leis de Conservação da Mecânica dos Fluidos aplicadas à Hidrodinâmica do Navio

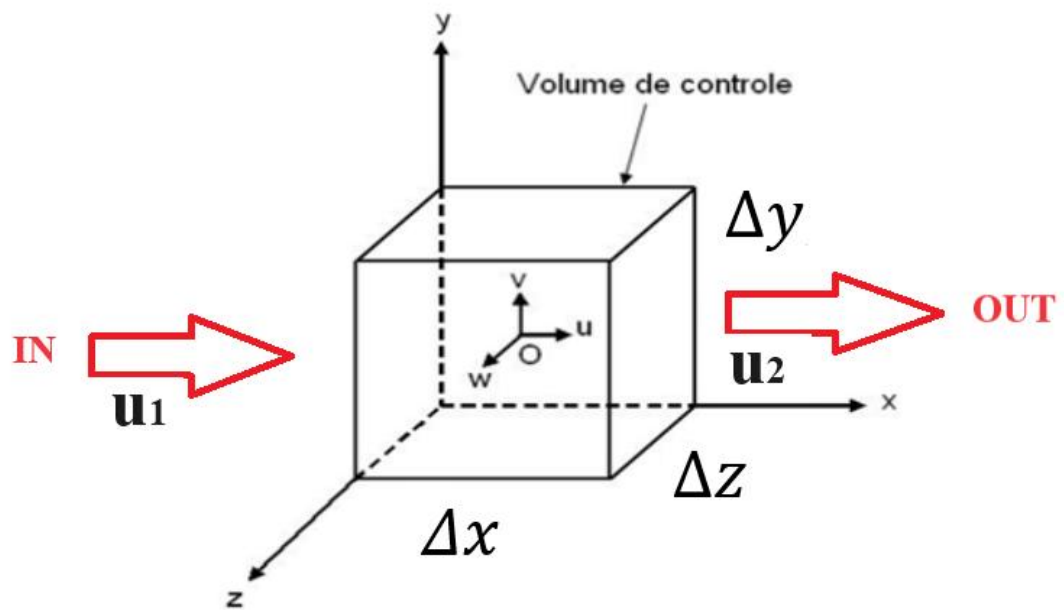
I – Lei da Conservação de Massa

Vol . . .

$p \quad \tau \quad - \quad -$

. . .

x 



$$p_{rxw} \quad p_l \quad p_{xer}$$

-

$$p \quad \tau x D$$

$$x \quad \frac{\tau}{w} \quad D \quad C \quad -$$

$$p_{rxw} \quad \tau x D$$

$$p_l \quad \tau x D$$

- -

$$p_{xer} \quad \frac{p}{w} \quad C \frac{\tau}{w} \quad - \quad -$$

$$\frac{\tau x}{}\quad \mathbb{G}\frac{\tau y}{}\quad \frac{\tau z}{}\quad \frac{\tau}{w}$$

$$\tau\qquad \frac{\tau}{w}$$

$$\quad \mathbb{G}\quad $$

$$u,v\,e\,w\qquad\qquad x,y\,e\,z$$

$$\tau\quad -\quad g_D\quad \frac{g}{g w}\quad \tau \mathbb{G} g_{ro}$$

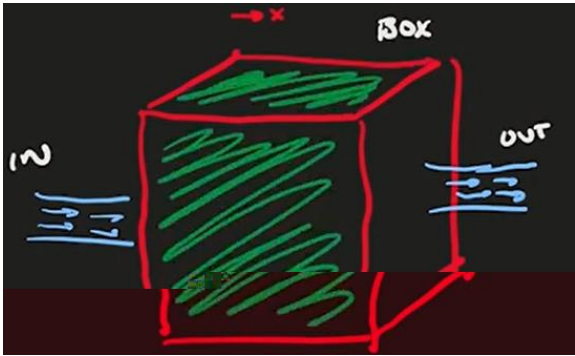
$$\frac{\tau}{w}$$

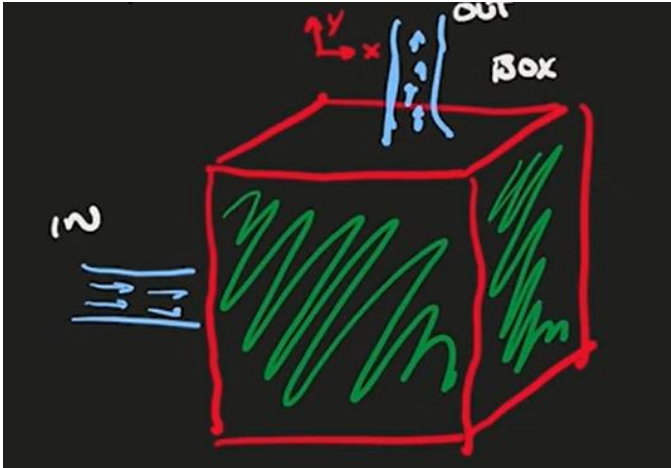
$$\qquad\qquad\qquad \tau$$

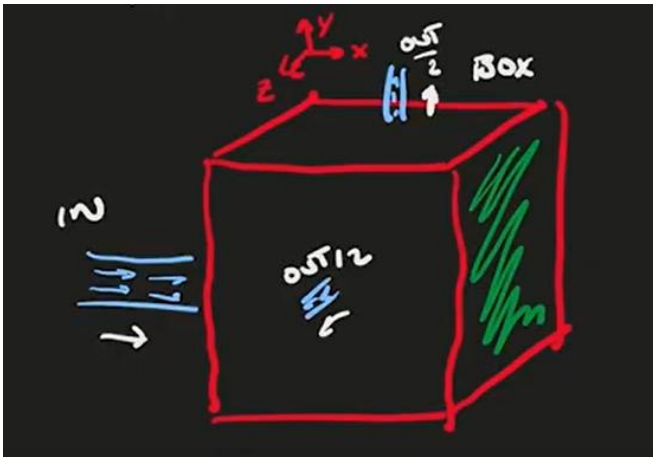
$$\frac{\tau x}{}\quad \mathbb{G}\frac{\tau y}{}\quad \frac{\tau z}{}\quad \tau\quad \frac{x}{}\quad \mathbb{G}\frac{y}{}\quad \frac{z}{}$$

$$\frac{x}{}\quad \mathbb{G}\frac{y}{}\quad \frac{z}{}$$

$$-$$







$$\underline{G^x} \quad \underline{G^y} \quad \underline{z}$$

II – A lei de conservação da quantidade de movimento

$$\frac{p}{w} = C$$

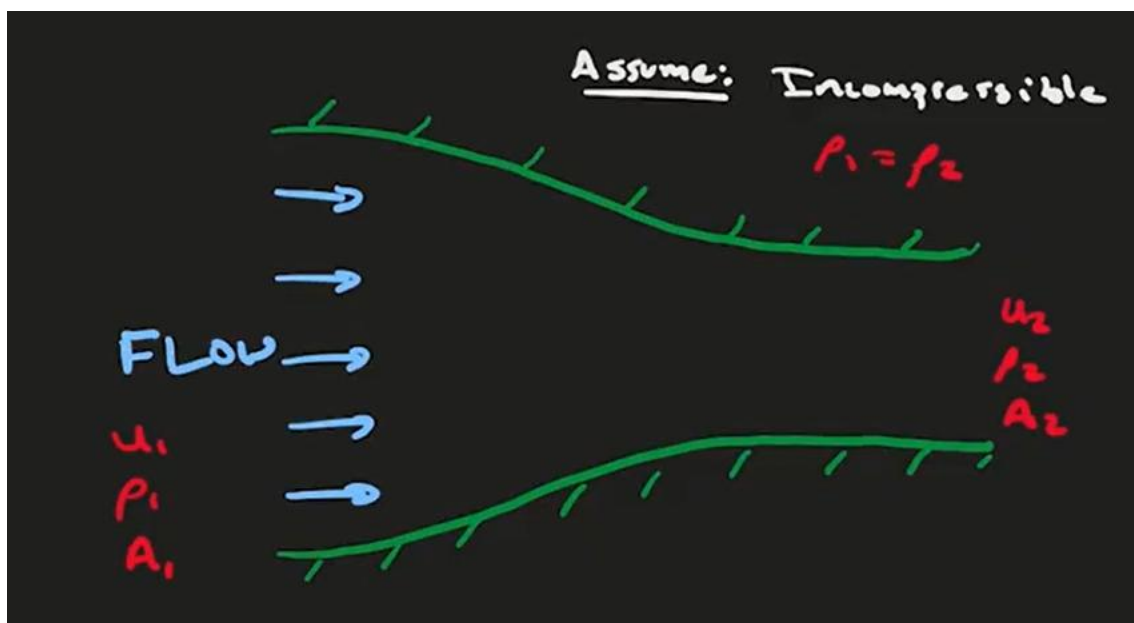
A derivada material, derivada substantiva ou derivada total

$$\tau \frac{dx}{dt}$$

$$(u \text{ e } u)$$

$$\tau \frac{dh}{dt}$$

$$(A \text{ e } A)$$



$$\tau \times D \qquad \tau \times D$$

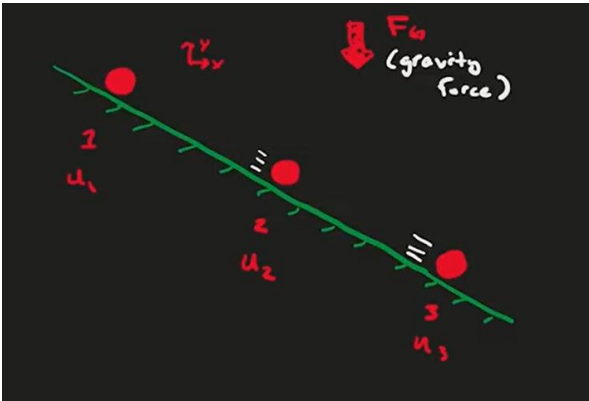
$$\tau \qquad \zeta \tau$$

$$x \, D \qquad x \, D$$

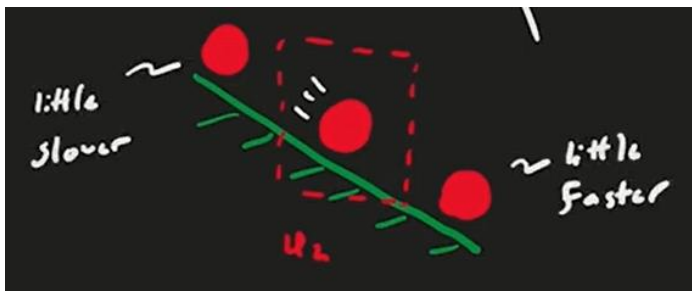
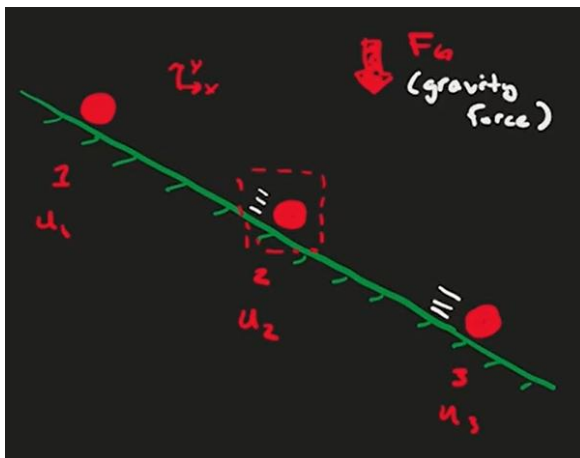
$$D \qquad D \qquad x \qquad x$$

$$\frac{x}{w}$$

$$\frac{x}{w}$$



$$x \qquad x \qquad x$$





$$\frac{x}{w} \quad \frac{x}{w} \quad \frac{x}{w} \quad x \frac{x}{w}$$

$$\frac{x}{w} \quad \frac{x}{w} \quad \frac{x}{w} \quad y \frac{x}{w}$$

$$C$$

$$d \quad \frac{x}{w} \quad x \frac{x}{w} \quad y \frac{x}{w} \quad z \frac{x}{w} C$$

DvGdfhdud hvGqdvGluh hvGhGhGhu rGhvs hf lydp hq hC

$$d \quad \frac{y}{w} \quad x \frac{y}{y} \quad y \frac{y}{y} \quad z \frac{y}{y} C$$

C

$$d \quad \frac{z}{w} \quad x \frac{z}{y} \quad y \frac{z}{y} \quad z \frac{z}{y} C$$

C

dvrGhvf rdp hq rGhndG hup dqhq hChp vhc

$$G \frac{x}{w} \quad .G \frac{y}{w} \quad ChG \frac{z}{w}$$

Forças atuantes em um elemento fluido

$$\frac{\frac{p}{o}}{w} \quad G \frac{o}{o}$$

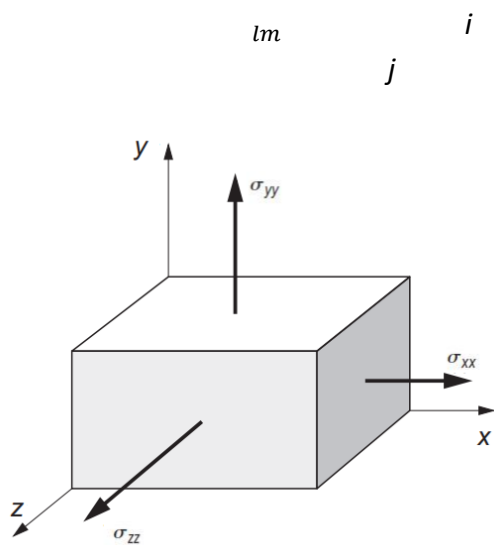
$$\frac{p}{ro} \quad \tau$$

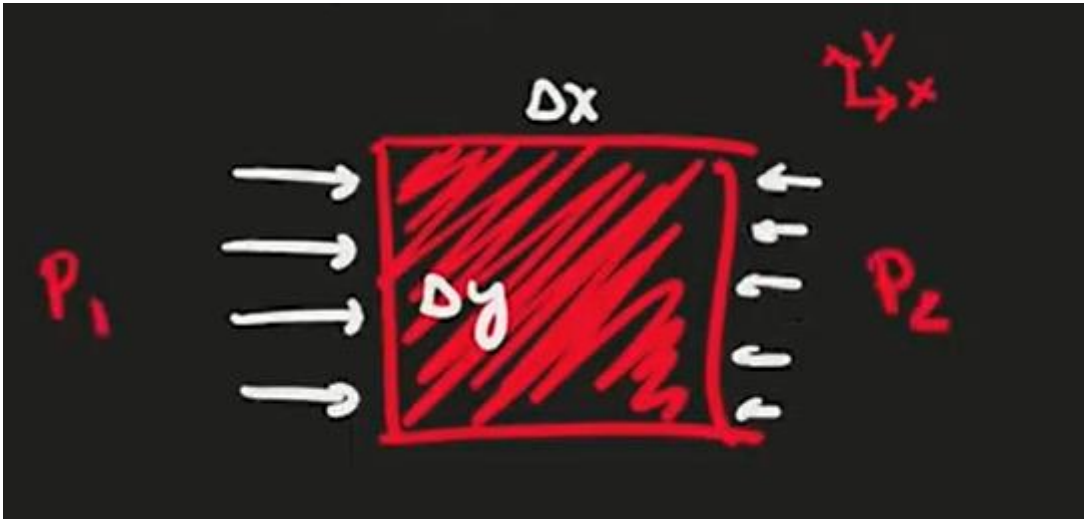
$$\frac{\tau}{w} \quad G \frac{o}{o}$$

$$\tau \frac{w}{w} \quad G \frac{o}{o}$$



As forças normais ou de pressão





$$s \quad \frac{G}{D} \quad G D$$

$$s \ D \quad s \ D$$

$$D \quad D$$

$$s \quad s$$

$$s \quad s \quad s$$

$$s$$

$$\frac{\tau}{r_0} = \frac{S}{r_0}$$

$$\frac{\tau}{r_0} = \frac{S}{r_0}$$

$$\frac{\tau}{r_0} = \frac{S}{r_0}$$

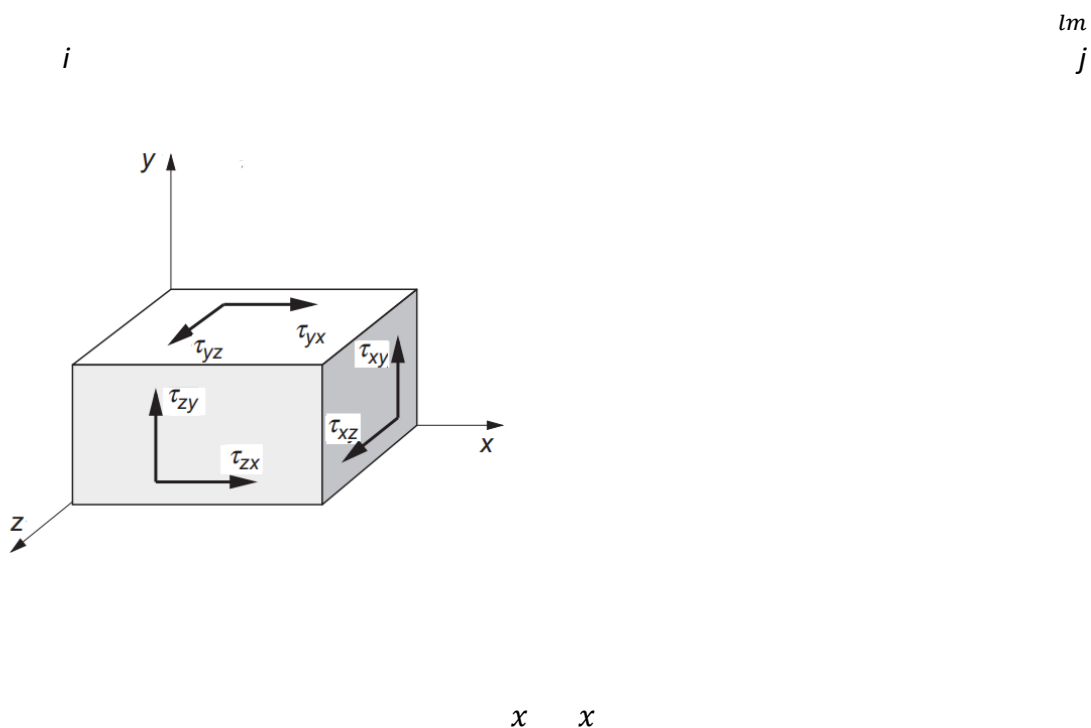
$$\frac{\tau}{r_0} = \frac{S}{r_0}$$

$$\tau \frac{x}{w} = \frac{S}{w}$$

$$\tau \frac{y}{w} = \frac{S}{w}$$

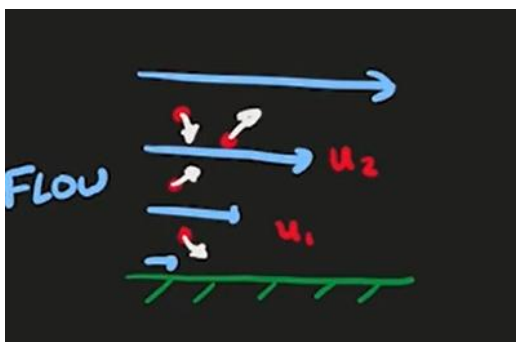
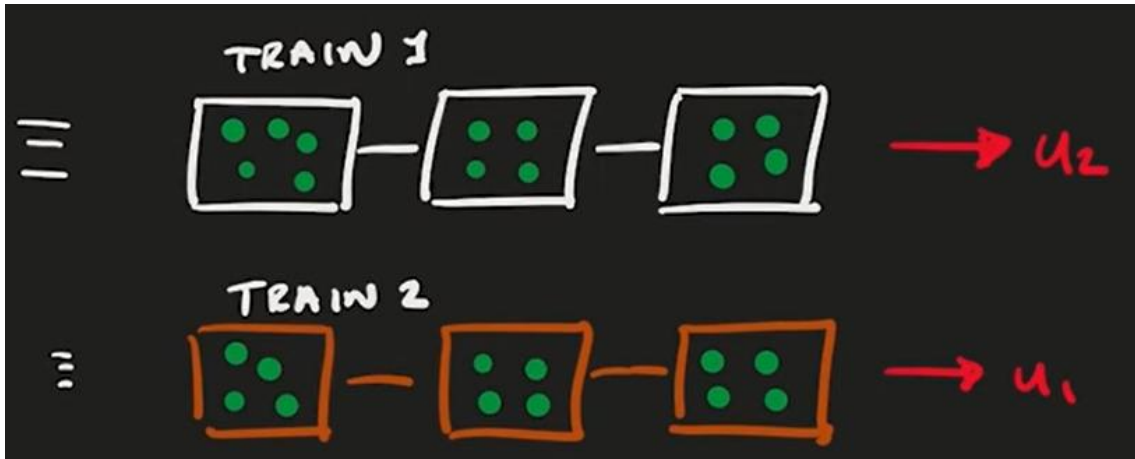
$$\tau \frac{z}{w} = \frac{S}{w}$$

As forças tangenciais, também conhecidas como forças friccionais ou viscosas



x

x



C_D

C_D

$$C_D = \frac{p}{D} \frac{x}{w}$$

$$\frac{x}{w}$$

—

$$\frac{x}{w} = \frac{x}{w} = \frac{x}{w}$$

$$\frac{p}{D} \frac{x}{w} = \frac{p}{D} \frac{x}{w}$$

$$\frac{p}{D} \frac{x}{w}$$

$$\frac{p}{D} \frac{x}{w}$$

$$\frac{x}{w}$$





D *D*

D *D*

x

x

x

o

x

o

x

x

x

$\tau \frac{x}{w}$

s

x

x

x

$$\tau \frac{y}{w} \quad \frac{s}{\quad} \quad \frac{y}{\quad} \quad \frac{y}{\quad} \quad \frac{y}{\quad}$$

$$\tau \frac{z}{w} \quad \frac{s}{\quad} \quad \frac{z}{\quad} \quad \frac{z}{\quad} \quad \frac{z}{\quad}$$

As forças de corpo

$$\mathbb{P} \quad \mathbb{G}$$

$$\text{—————} \quad \mathbb{G}$$

$$\tau \frac{x}{w} \quad \frac{s}{\quad} \quad \frac{x}{\quad} \quad \frac{x}{\quad} \quad \frac{x}{\quad} \quad \tau$$

$$\tau \frac{x}{w} \quad x \frac{x}{\quad} \quad y \frac{x}{\quad} \quad z \frac{x}{\quad} \quad \frac{s}{\quad} \quad \frac{x}{\quad} \quad \frac{x}{\quad} \quad \frac{x}{\quad} \quad \tau \quad C$$

hGr up dGlp lduGdvGluh hvôôôGhu rGhvs hf lydp hq hC

$$\tau \frac{\quad}{w} \quad x \frac{y}{\quad} \quad y \frac{y}{\quad} \quad z \frac{y}{\quad} \quad \frac{s}{\quad} \quad \frac{y}{\quad} \quad \frac{y}{\quad} \quad \frac{y}{\quad} \quad \tau \quad C$$

C

$$\tau \frac{z}{w} \quad x \frac{z}{\quad} \quad y \frac{z}{\quad} \quad z \frac{z}{\quad} \quad \frac{s}{\quad} \quad \frac{z}{\quad} \quad \frac{z}{\quad} \quad \frac{z}{\quad} \quad \tau \quad C$$

C

DvC u vChtxd hvCdflp dCfruhvsrqghp CdGdp rvdC tx rCghCQdylhu V rnhv.C xhCqdC
irup dCgh ruldC Cdsuhvhq dgdC ruC

$$\tau \frac{\quad}{w} \quad s \quad \tau \quad C$$

C

C DChlCghCfrqvhuyd rCghCqhuji dC

Srgh vCgylgluCdChqhuji dCghCxp Cvlv hp dChp CgxvCfadvvhvCglv lq dvCdChqhuji dC
dup ddhqdgdChCdChqhuji dChp C udqvl rCDsulp hludChv CuhadflrqdgdCgluh dp hq hC
uhadflrqdgdCfrp Cdp dvvdCgrC su s ulrGlv hp d.ChCqyrqyhCdvChqhuji dvlq huqd.Cflq lfdC
hCs r hqfidoCdCvhj xqgdCfadvvhCvCuhadflrqdCfrp CdChqhuji dC urfdgdChq uhCvlv hp dv.C
lqfaklqgrGrCfdruCGrCude dkrC r qvlghudqgrC CdChqhuji dCr dddup ddhqdgdC ruCxp C
vlv hp dC xhCxp Cq huydrCghCp sr.CghfhehxGp dC xdq lgdghCghCfdruC Cgh hf x rxC
xp Cude dkrC .Cdf r qvhuyd rCghvvhGlv hp dC CudgxdlgdC udy vCgdCh suhvv rC

C

DCh suhvv rCdflp dC Cdc sulp hludChlCgdChup rglq p lfd C

q r.CdC d dCghCyduld rCgdChqhuji dCs ruCxlgdghCghC hp srC $\frac{g}{gw}$.Cd vC urfdvCghC
Hqhuji dChih xdgCfrp GrC hulruGreGrup dCghCfdruC $\frac{g}{gw}$.ChCdCghC ude dkr. $\frac{g}{gw}$
CuhvxodqgrChp C

$$\frac{g}{gw} \quad C$$

C xhGvj qlf dCf dhuC xhCdC dCghCq udgd.C ruCxlgdghCghCp sr.CghCqhuji dC up lfdC
hCp hf qlfdCxp Glv hp dCp hqr vCfruhvsrqghq hCd dCghCd gdCghyhu Gj xdaduCdC dC
ghCf xp xad rCghCqhuji dCqrCq hulruCghvvhCp hvp rGlv hp dC

SdudCuhadflrqduhv hvCfrqfhl rvdCf lp dCfrp Cdq r rCghCyræp hCghCfrq urdChp suhj d
vhCfrqfhl rCghCghqvlgdghCp vvlfdCghCqhuji dCdup ddhqdgdC horGlv hp d.C

h

μ

$$\frac{g}{g_w} = \frac{h\tau C_{gy}}{w} - h\tau - g_{DC}$$

$$p$$

$$h$$

$$p \quad h$$

$$h$$

$$h \quad s_{uhvv} \, r \quad ylv \, rvdv$$

$$h$$

$$p$$

$$s_{uhvv} \, r$$

$$gD$$

$$g \, s_{uhvv} \, r \quad s \quad - \quad gD$$

$$s_{uhvv} \, r \quad s \quad - \quad gD$$

$$\gamma \rho v dv$$

$$\gamma \int \rho v dv = - \gamma D$$

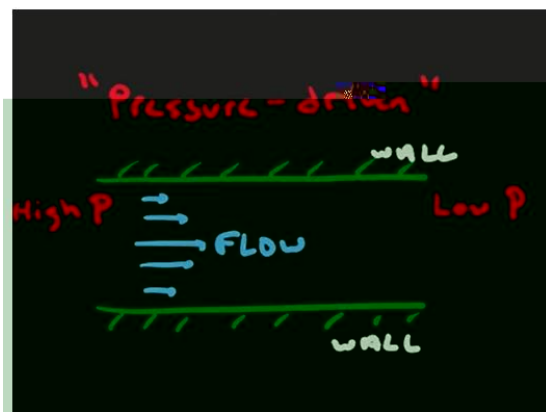
$$\gamma \rho v dv = - \gamma D$$

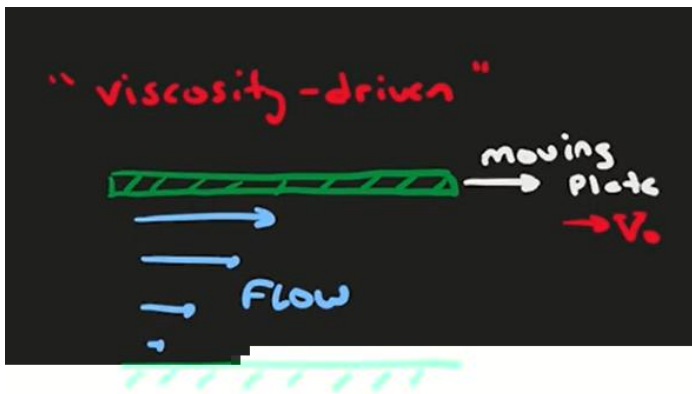
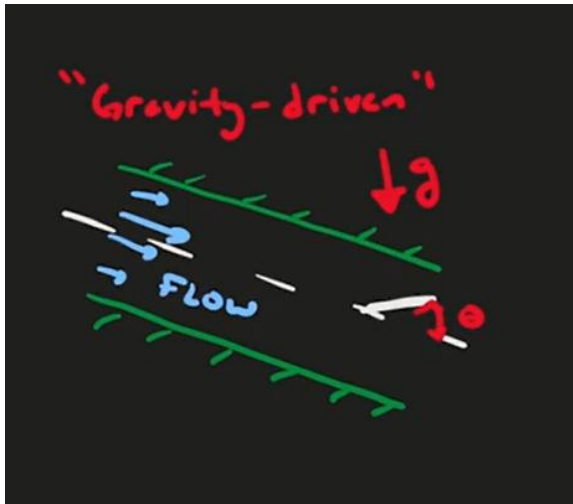
$$p + \gamma s = - \gamma D = - \gamma D$$

$$\frac{1}{\rho} \frac{d\rho}{dx} = - \frac{\tau}{\rho g y} \quad \text{ou} \quad \frac{1}{\rho} \frac{d\rho}{dx} = - \frac{s}{\rho g D}$$

Hipóteses comum na mecânica dos fluidos

Aplicações simples das leis de conservação de massa e de conservação de quantidade de movimento - Escoamentos laminares internos fechados





Escoamento plenamente desenvolvido dominado pelas forças de pressão

s

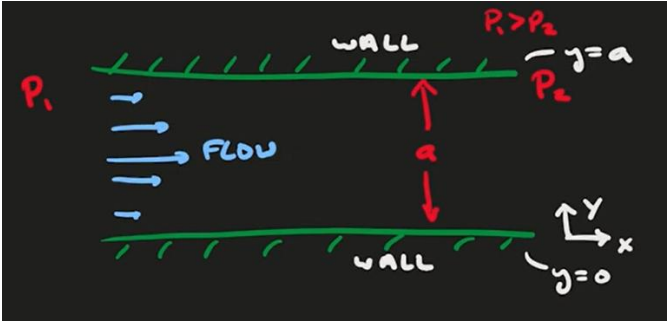
s

s

s

d

d



$$\tau \quad \nu w$$

$$\frac{\tau}{w}$$

$$\frac{x}{z}$$

$$\tau$$

$$\frac{G}{z} \quad \frac{z}{d}$$

$$\frac{x}{z} \quad \frac{x}{d}$$

$$\frac{y}{z} \quad \frac{y}{d}$$

$$\frac{x}{G} \quad \frac{y}{z}$$

$$\frac{x}{z}$$

$$\frac{z}{y}$$

$$\frac{y}{z}$$

$$\frac{y}{-g} \quad y \quad vw \quad F$$

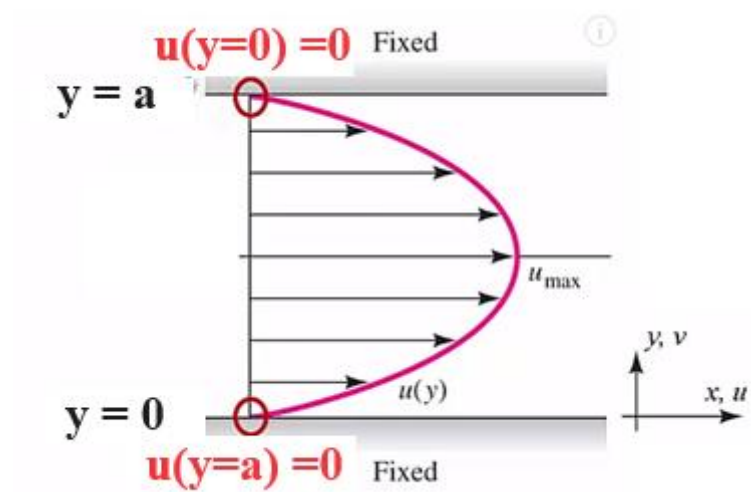
$$x_3 \quad x_d$$

$$x_3 \quad -\frac{s}{F} \quad F \quad F$$

$$x_d \quad -\frac{s}{F} d \quad F d$$

$$x \quad -\frac{s}{F} \quad -\frac{s d}{F}$$

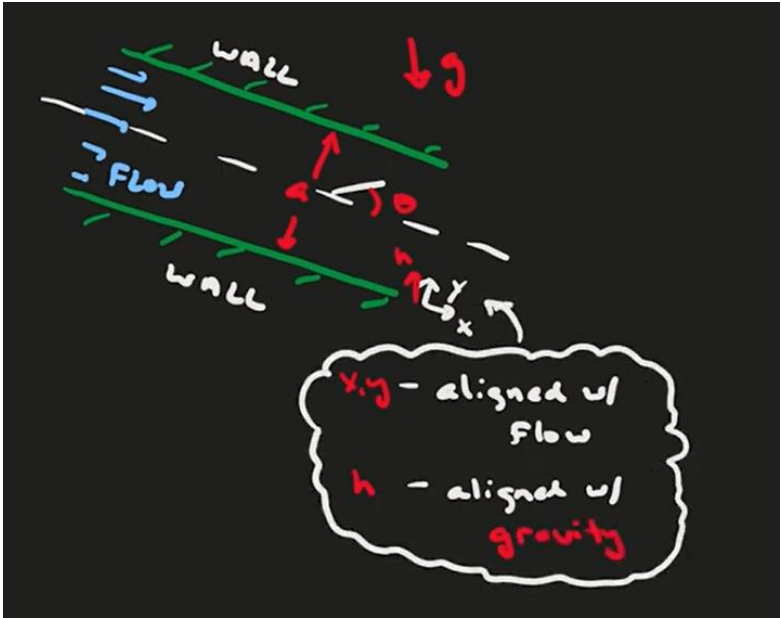
$$x \quad -\frac{s}{F} \quad d$$



Escoamento plenamente desenvolvido dominado pela força de corpo

d

d



τ νw

$$\frac{\tau}{w}$$

$$\frac{x}{z}$$

$$\frac{\tau}{z}$$

$$x_3 \quad x_d$$

$$z_3 \quad z_d$$

$$y_3 \quad y_d$$

$$\frac{x}{}\quad \mathbb{G}\frac{y}{}\quad \frac{z}{}$$

$$\frac{x}{}$$

$$\frac{z}{}$$

$$\frac{y}{}$$

$$\frac{y}{}g\quad y\quad vw\quad F$$

$$y\quad 3.d$$

$$F$$

$$y$$

$$\tau\quad \frac{x}{w}\quad x\frac{x}{}\quad y\frac{x}{}\quad z\frac{x}{}\quad \frac{s}{}\quad \frac{x}{}\quad \frac{x}{}\quad \frac{x}{}\quad \tau\quad C$$

$$\frac{x}{w}$$

$$x\frac{x}{}\quad x\frac{x}{}$$

$$z\frac{x}{}\quad \frac{x}{}$$

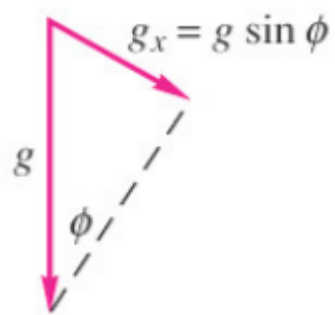
$$\tau$$

$$y\quad y\frac{x}{}$$

$$\frac{s}{}\quad \frac{x}{}\quad \tau$$

$$\frac{x}{}\quad \frac{s}{}\quad \tau$$

$$\mathfrak{vlq}$$



$$v_{\text{q}} = \frac{g}{g}$$



$$\frac{x}{s} = \frac{\tau}{v_w}$$

$$\frac{x}{s} = \frac{\tau}{F}$$

x

$$x = \frac{s}{F} \tau$$

$$x_3 = x_d$$

$$x_3 = \frac{s}{F} \tau_j$$

$$F$$

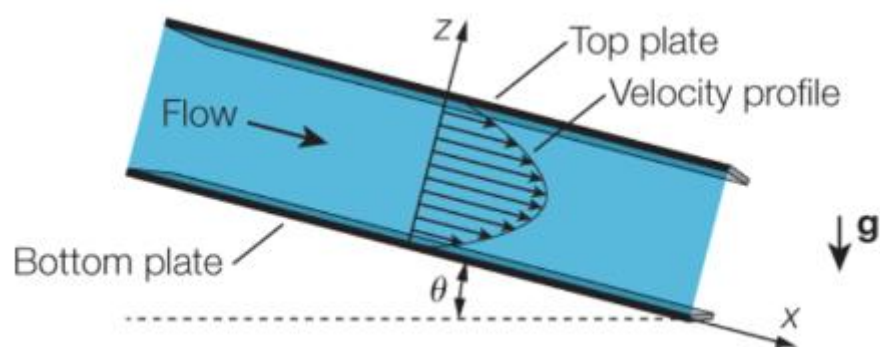
$$x \quad d \quad - \frac{s \tau_j d}{F} \quad F d$$

$$F \quad - \frac{s \tau_j d}{F}$$

$$F \quad F$$

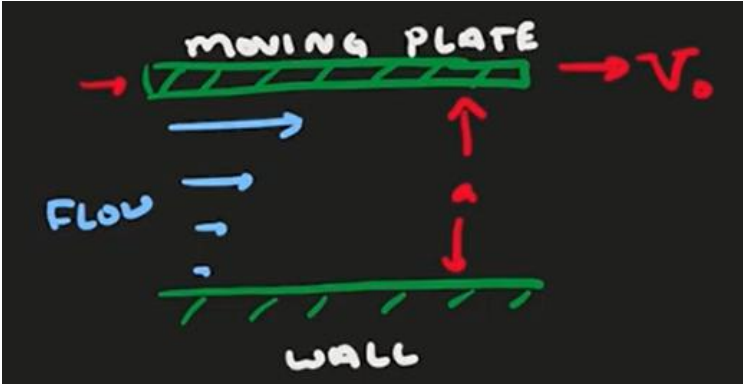
$$x \quad - \frac{s \tau}{d} \quad - \frac{s \tau d}{d}$$

$$x \quad - \frac{s \tau}{d} \quad d$$



Escoamento plenamente desenvolvido dominado pelas forças friccionais

$$s \quad V \quad s \quad s \quad s \quad d$$



$$\tau \quad vw$$

$$\frac{-}{w}$$

$$\frac{x}{z}$$

$$\frac{-}{z}$$

$$\frac{z}{d} \quad \frac{z}{d}$$

$$\frac{x}{d} \quad \frac{x}{d}$$

$$\frac{y}{d} \quad \frac{y}{d}$$

$$\frac{x}{G} \quad \frac{y}{G} \quad \frac{z}{G}$$

$$\frac{x}{z}$$

$$\frac{z}{z}$$

$$\frac{y}{y}$$

$$\frac{y}{g} \quad y \quad vw \quad F$$

$$y_{3.d}$$

$$F$$

$$y$$

$$\tau \frac{x}{w} \quad x \frac{x}{} \quad y \frac{x}{} \quad z \frac{x}{} \quad \frac{s}{} \quad \frac{x}{} \quad \frac{x}{} \quad \frac{x}{} \quad \tau \quad C$$

$$\frac{x}{w}$$

$$x \frac{x}{} \qquad x \frac{x}{}$$

$$z \frac{x}{} \qquad \frac{x}{}$$

$$\tau$$

$$y$$

$$y \frac{x}{}$$

$$\frac{x}{}$$

$$\frac{x}{} \quad F$$

$$x$$

$$x \quad F \quad F$$

$$x_{3} \qquad x_{d} \qquad 3$$

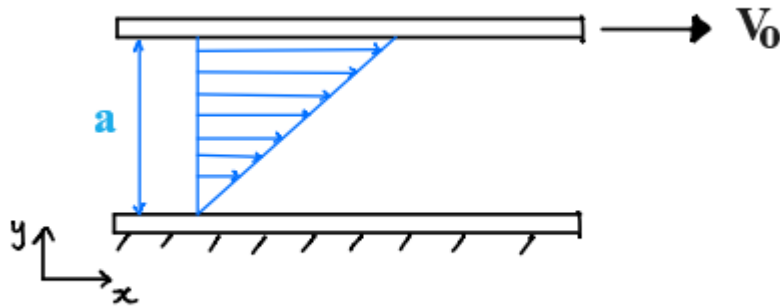
$$x_{3} \quad F \quad F$$

$$F$$

$$x_{d} \quad F \, d \qquad 3$$

$$F \quad \frac{}{d}$$

$$x \quad \frac{3}{d} \quad d$$



A Brief Side Comment: Developing Flow

- The velocity profile is initially uniform at the inlet.
- *Boundary layers* grow on both walls due to viscous drag.
- Boundary layers merge on the centre line.
- Velocity profile stops changing in x-direction, $\frac{\partial u}{\partial x} = 0$

Current exact solution is for flow that is fully developed, $\frac{\partial u}{\partial x} = 0$

